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# Initial state effects on the cosmic microwave background and trans-planckian physics

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**Abstract**

There exist a one complex parameter family of de Sitter invariant vacua, known as $\alpha$ vacua. In the context of slow roll inflation, we show that all but the Bunch-Davies vacuum generates unacceptable production of high energy particles at the end of inflation. As a simple model for the effects of trans-planckian physics, we go on to consider non-de Sitter invariant vacua obtained by patching modes in the Bunch-Davies vacuum above some momentum scale $M_c$, with modes in an $\alpha$ vacuum below $M_c$. Choosing $M_c$ near the Planck scale $M_{pl}$, we find acceptable levels of hard particle production, and corrections to the cosmic microwave perturbations at the level of $HM_{pl}/M_c^2$, where $H$ is the Hubble parameter during inflation. More general initial states of this type with $H \ll M_c \ll M_{pl}$ can give corrections to the spectrum of cosmic microwave background perturbations at order 1.

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## 1. Introduction

Inflation magnifies quantum fluctuations at fundamental length scales to astrophysical scales, where their imprint is left on the formation of structure in the universe. In conventional slow roll inflation, the universe undergoes an expansion of at least $10^{26}$ during the inflationary phase. With such a huge expansion factor, modes which give rise to observable structures apparently started out with wavelengths much smaller than the Planck length. This is the so-called trans-planckian problem in inflation [1–2].

In the past year, there has been much debate about whether potential modifications to physics above the Planck scale could actually be observed [6–8]. By considering the local effective action at the scale Hubble scale $H$ (which we will take to be $10^{13} - 10^{14}$ GeV), [13] has argued that trans-planckian corrections to the spectrum of cosmic microwave background perturbations could at best be of order $(H/M_{pl})^2$ which is typically far too small to be observed in conventional inflationary models. However others [6–8] have obtained a correction of order $H/M_{pl}$ by considering a variety of methods for modeling trans-planckian effects. Such a correction is potentially observable in the not too distant future.

In the present work we represent the effect of trans-planckian physics simply by allowing for nontrivial initial vacuum states for the inflaton field, which we treat as a free scalar field moving in a de Sitter background. The most natural vacuum states to consider are the de Sitter invariant vacuum states constructed by Allen and Mottola [15,16]. The vacuum states are known as $\alpha$-vacua. We find these all lead to infinite energy production at the end of inflation, with the exception of the Bunch-Davies (Euclidean) vacuum state.

We go on to consider non-de Sitter invariant vacuum states constructed by placing modes with comoving wavenumber $k > M_c a(\eta_f)$ in the Bunch-Davies vacuum, where $a(\eta_f)$ is the expansion factor at the end of inflation. Modes with $k < M_c a(\eta_f)$ are placed in a non-trivial $\alpha$ vacuum. These states have a particularly simple evolution in de Sitter space – the length scale at which the patching occurs simply expands as the scale factor grows. Many more complicated initial states asymptote to such states as the universe expands.

For $M_c$ of order $M_{pl}$ it is possible to find initial states that do not overproduce hard particles, and produce corrections to the cosmic microwave background spectrum at order $H/M_{pl}$ in agreement with [7]. For $H \ll M_c \ll M_{pl}$ there are initial states that produce corrections to the spectrum at order 1.

In [10], an initial state was constructed by placing modes in their vacuum states as the proper wavenumber passed through the scale of new physics $M_c$. This turns out to be a special case of the class of initial states we consider. To avoid large backreaction problems in this case, we show the condition $M_c \ll M_{pl}$ must hold. This condition is rather easy to satisfy.

## 2. General setup

We will conduct our analysis using linearized perturbation theory in a de Sitter (dS) background. Planar coordinates covering half of dS, with flat spacial sections, result in the metric

$$ds^2 = dt^2 - e^{Ht}d\vec{x}^2 = dt^2 - a^2(t)d\vec{x}^2 \ . \tag{2.1}$$

It will be more convenient to use conformal coordinates, giving

$$ds^2 = \frac{1}{(\eta H)^2}\left(d\eta^2 - d\vec{x}^2\right) = a^2(\eta)\left(d\eta^2 - d\vec{x}^2\right) \tag{2.2}$$

where $\eta = \int_t^\infty dt'/a(t') = -\exp(-Ht)/H$. So $t \to -\infty$ and $\eta \to -\infty$, and $t \to \infty$ as $\eta \to 0$.

Klein-Gordon Equation in curved space is

$$(\Box + m^2 + \zeta R)\phi = 0 \tag{2.3}$$

for a scalar field with mass $m$ and non-minimal coupling to $R$ given by $\zeta$. In momentum space we can solve this equation by defining

$$\phi_k = \frac{e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{\frac{3}{2}}a(\eta)}\chi_k(\eta) \tag{2.4}$$

which leads to

$$\chi_k'' + \left(k^2 + \frac{M^2}{H^2\eta^2}\right)\chi_k = 0 \tag{2.5}$$

with

$$M^2 = m^2 + (\zeta - \frac{1}{6})R \tag{2.6}$$

so $M^2$ is not necessarily positive. The general solution is

$$\chi_k(\eta) = \tfrac{1}{2}\sqrt{\pi\eta}\ H_\nu^{(2)}(k\eta) \equiv \chi_{Ek}(\eta) \tag{2.7}$$

together with its complex conjugate, where $\nu = \frac{9}{4} - \frac{m^2}{H^2} - 12\zeta = \frac{1}{4} - M^2$.

Such a complete set of orthonormal modes may be used to define a Fock vacuum state by taking the field operator

$$\Phi = \sum_k \chi_k(\eta) a_k + \chi_k^*(\eta) a_{-k}^\dagger \tag{2.8}$$

and demanding $a_k|0\rangle = 0$. As shown by Allen [15] and Mottola [16], the general family of de Sitter invariant vacuum states can be defined using the modes

$$\chi_k = \cosh\alpha \ \chi_{Ek}(\eta) + e^{i\delta} \sinh\alpha \ \chi_{Ek}^*(\eta) \tag{2.9}$$

with $\alpha \in [0, \infty)$ and $\delta \in (-\pi, \pi)$. $\alpha = 0$ is the Bunch-Davies vacuum (a.k.a. Euclidean vacuum).

For a massless minimally coupled scalar, this solution takes a particularly simple form

$$\chi_{Ek}(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}}(1 - \frac{i}{k\eta}) \ . \tag{2.10}$$

As discussed in [15] this case gives rise to difficulties in canonical quantization, and there is no de Sitter invariant Fock vacuum. Nevertheless, we will use this simple example in the following with the understanding a small mass term could be added to eliminate this problem, and the expressions we derive will not be substantially changed.

We will need to extract two physical quantities from the expression (2.9). The first is the number of particles produced in the mode $k$ defined with respect to the $\alpha = 0$ vacuum. This is simply equal to

$$n_k = \sinh^2\alpha \ . \tag{2.11}$$

This will be a good approximation to the number of particles produced at the end of inflation, when a transition is made to a much more slowly expanding universe, provided the wavelength of the modes in question are much smaller than the Hubble radius. This follows simply from the fact that at high wavenumber, the wave equation for $\chi_k$ reduces to that of flat space, so we can approximate the final geometry by Minkowski space. We wish to count particles with respect to the Lorentz invariant vacuum state, which corresponds to the $\alpha = 0$ vacuum in this regime.

The second physical quantity of interest is the contribution of this mode to the spectrum of CMBR perturbations. We compute this by examining $|\phi_k(\eta)|^2$ in the distant future $\eta \to 0$ for the massless scalar (2.10). The contribution is then

$$P_k = \frac{k^3}{2\pi^2 a^2}|\chi_k|^2 = \left(\frac{H}{2\pi}\right)^2 |\cosh\alpha - e^{i\delta}\sinh\alpha|^2 \ . \tag{2.12}$$

## 3. Initial state effects

We begin by reviewing what happens for the usual Bunch-Davies vacuum, $\alpha = 0$. Clearly the particle production at high frequencies (2.11) vanishes. Fluctuations in the scalar field modes mean different regions of spacetime expand at slightly different rates, which gives rise to density perturbations after inflation has ended. The amplitude of these perturbations are frozen in as these modes expand outside the Hubble radius during inflation, and become density perturbations once they reenter the horizon after the end of inflation. For $\alpha = 0$, $P_k = (H/2\pi)^2$ is independent of $k$ and hence scale invariant. When one allows for the detailed shape of the inflaton potential, $H$ becomes effectively $k$ dependent, leading to small deviations from the scale invariant spectrum of perturbations, which in general are highly model dependent.

For a nontrivial $\alpha \neq 0$ vacuum we immediately see a problem. At the end of inflation there will be a large amount of particle production at wavelengths smaller than the Hubble radius (2.11). Since this production is independent of $k$, this will lead to an infinite energy density, which will cause the universe to recollapse. Clearly this is very different to our own universe. We conclude then that at wavelengths below some scale, the modes must be in a local $\alpha = 0$ vacuum state.

Nevertheless, we can still consider initial states that involve modes in an $\alpha \neq 0$ state, provided their wavelengths are sufficiently large. Perhaps the simplest such initial state is to place modes at some fixed conformal time $\eta_0$ in the $\alpha = 0$ state for $k > M_c a(\eta_f)$ where $\eta_f$ is the conformal time at the end of inflation, and $M_c$ is some scale at which physics changes, and we have in mind taking $M_c \gg H$. Modes for $k < M_c a(\eta_f)$ can be placed in an $\alpha \neq 0$ state.

In order that the particle production at the end of inflation be irrelevant versus the energy stored in the inflaton, we must have

$$M_c^4 \sinh^2 \alpha \ll \Lambda = \frac{3M_{pl}^2 H^2}{4\pi} \tag{3.1}$$

where $M_{pl}$ is the Planck mass.[1] If we saturate this bound, $\sinh \alpha \sim HM_{pl}/M_c^2$. The correction to the CMBR spectrum $P_k$ (2.12) will then be of order $HM_{pl}/M_c^2$. This is

[1] This condition is necessary to avoid large back-reaction on the geometry. It would also be interesting to consider the limit when this energy is not irrelevant, and to use this particle production as a source for reheating.

linear in $H$ in agreement with the estimates of [6,7,10] and is potentially observable. Of course, since we have done the computation in pure de Sitter space, the effect appears as a $k$-independent modulation of the $\alpha = 0$ result, which on its own would require an independent determination of $H$ to measure directly. However, in inflation $H$ is actually slowly changing, which will translate into k-dependence of $H$, and hence $\alpha$. This will show up as $k$-dependent corrections to the cosmic microwave background spectrum $P_k$ which are potentially more easily distinguishable from the $\alpha = 0$ case [6].

To obtain an upper bound on the size of the correction to the CMBR spectrum, we can imagine taking $M_c$ to be much smaller than $M_{pl}$, which is certainly plausible. This allows $\alpha$ to be of order 1, and still consistent with negligible hard particle production (3.1). This limit will lead to corrections to the CMBR spectrum (2.12) at order 1.

### *3.1. Transition at proper energy $M_c$*

Now let us consider a more detailed model for the initial state where we assume the initial condition is fixed due to some change in physics at the proper energy scale $M_c$. Let us review the computation of [10]. To set up this calculation one works in the Heisenberg picture, where time dependent field operators $\mu_k(\eta)$ (here $\mu_k$ solves the wave equation (2.5) with $m^2 = 0$, $\zeta = 0$) may be expressed in terms of operators at some fixed time $\eta_0$ as

$$\mu_k(\eta) \equiv \frac{1}{\sqrt{2k}}(a_k(\eta) + a^\dagger_{-k}(\eta)) = f_k(\eta) a_k(\eta_0) + f^*_k(\eta) a^\dagger_{-k}(\eta_0) \tag{3.2}$$

where $f_k$ is some general solution to the wave equation

$$f_k(\eta) = A_k \; \chi_{Ek}(\eta) + B_k \; \chi^*_{Ek}(\eta) \; . \tag{3.3}$$

A similar expression to (3.2) holds for the conjugate momentum.

The creation/annihilation operators at time $\eta$ are related by a Bogoliubov transformation to those at a different time $\eta_0$

$$\begin{aligned} a_k(\eta) &= u_k(\eta) a_k(\eta_0) + v_k(\eta) a^\dagger_{-k}(\eta_0) \\ a^\dagger_{-k}(\eta) &= u^*_k(\eta) a^\dagger_{-k}(\eta_0) + v^*_k(\eta) a_k(\eta_0) \end{aligned} \tag{3.4}$$

with

$$f_k(\eta) = \frac{1}{\sqrt{2k}}(u_k(\eta) + v^*_k(\eta)) \; . \tag{3.5}$$

One then defines the initial state by demanding that the mode $k$ satisfy

$$a_k(\eta_0)|\alpha, \eta_0\rangle = 0 \ . \tag{3.6}$$

Using the Bogoliubov transformation (3.4), (3.5) and the analogous relation for the conjugate momentum, this in turn implies

$$B_k = \frac{ie^{-2ik\eta_0}}{2k\eta_0 + i} A_k \ . \tag{3.7}$$

Since the proper wavenumber of each mode changes with time, we want to choose $\eta_0 = -M_c/Hk$, so the boundary condition is imposed when the proper wavenumber is $M_c$. Thus the relation between $A_k$ and $B_k$ is independent of $k$, and this mode is in an $\alpha$ vacuum, with $\alpha$ and $\delta$ determined by $H/M_c$ [10,12]. For sufficiently large $k$, the above prescription (3.6) does not apply, because the relevant $\eta_0$ will be after the end of inflation. These modes may safely be placed in the Bunch-Davies vacuum.

This initial state is a special case of the type described above. High frequency particle creation gives an energy density of order $M_c^4 \sinh^2 \alpha$. Since here $\sinh \alpha \sim H/M_c$, we require $M_c^2 H^2 \ll M_{pl}^2 H^2$. This will hold whenever $M_c \ll M_{pl}$, which is easy to satisfy.

## 4. Conclusions

We have constructed a very simple class of initial states for the inflaton field which can be used to model effects of trans-planckian physics. For certain ranges of parameters, these states do not lead to excess particle production at the end of inflation, and lead to potentially observable corrections to the cosmic microwave background spectrum.

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